Lorentz-Covariant Quantization of Massless Non-Abelian Gauge Fields in The Hamiltonian Path-Integral Formalism

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Abstract

The Lorentz-covariant quantization performed in the Hamiltonian path-integral formalism for massless non-Abelian gauge fields has been achieved. In this quantization, the Lorentz condition, as a constraint, must be introduced initially and incorporated into the Yang-Mills Lagrangian by the Lagrange undetermined multiplier method. In this way, it is found that all Lorentz components of a vector potential have thier corresponding conjugate canonical variables. This fact allows us to define Lorentz-invariant poisson brackets and carry out the quantization in a Lorent-covariant manner.

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one knows, the massless non-Abelian gauge fields was first quantized by the elegant Faddeev-Popov approach[1]. This approach works in the Lagrangian (or say, the second order)path-integral formalism. Subsequently, it was shown that the gauge fields may also be quantized in the perfect Hamiltonian (or say, the first order)path-integral formalism[2]−[5] along the line of quantization proposed first by Dirac for constrained systems[6]. The Hamiltonian path-integral quantization usually is performed in the coulomb

gauge[2]−[5]. Therefore, the quantization and quantized result are Lorentz-non-covariant. However, the Lorentz-covariant form of the quantum theory as obtained by the Faddeev-Popov approach is mostly used in practical applications. Whether and how the massless non-Abelian gauge field can be Lorentz-covariantly quantized in the Hamiltonian path-integral formalism? This just is the question we try to answer in this paper.

Let us recast the Yang-Mills Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} \tag{1}$$

in the first order form[2].

$$\mathcal{L} = E^{ak} \dot{A}^a_k + A^a_0 C^a - \mathcal{H} \tag{2}$$

where E^{ak} is defined by

$$E^{ak} = \frac{\partial \mathcal{L}}{\partial \dot{A}^a_k} = F^{ako}, k = 1, 2, 3. \tag{3}$$

which is the field momentum density conjugate to the coordinate A^a_k,

$$C^a = \partial^k E^a_k + g f^{abc} A^{bk} E^c_k \tag{4}$$

and

$$\mathcal{H} = \frac{1}{2}(E^a_k)^2 + \frac{1}{2}(B^a_k)^2 \tag{5}$$

here

$$B^a_k = -\frac{1}{2} \in_{kij} F^a_{ij} \tag{6}$$

$\mathcal{H}$ is the Hamiltonian density of the field. In the above, we have used the Greek letters to denote the four-dimensional indices and the Latin letters to mark the three-dimensional indices. From the stationary condition of the action given by the Lagrangian in Eq.(2), it is easy to derive the following equations of motion[2],[3]

$$\dot{A}^a_k - \partial_k A^a_0 + g f^{abc} A^b_0 A^c_k + E^a_k = 0 \tag{7}$$

$$\dot{E}^a_k + \partial^l F^a_{kl} + g f^{abc} A^b_0 E^c_k + g f^{abc} A^{bl} F^c_{kl} = 0 \tag{8}$$

$$C^a \equiv \partial^k E_k^a + gf^{abc} A_k^b E^{ck} = 0 \tag{9}$$

The last equation (9) is identified with a constraint condition because there is no time-derivatives of the field variables in it. This constraint condition, as shown in the second term in Eq.(2), has already been incorporated into the Lagrangian by the Lagrange undetermined multiplier method so as to make the Lagrangian written in Eq.(1) or (2) to be Lorentz-invariant. The function A_0^a in Eq.(2) acts as a Lagrange multiplier. Since a massless gauge field has only two polarization states, among the three pairs of canonical variables (A_k^a, E_k^a) for a given group index a, only two pairs can be viewed as the independent dynamical variables. Therefore, in addition to the constraint in Eq.(9), it is necessary to introduce another constraint so as to eliminate the redundant degrees of freedom appearing in the theory. There are various choices of the constraint which are physically equivalent. Commonly, the Coulomb gauge condition

$$\partial^k A_k^a = 0 \tag{10}$$

is preferable to be chosen[2]−[5]. The necessity of introducing an additional constraint implies that the Yang-Mills Lagrangian in Eq.(1) itself can not give a complete description of the massless gauge field dynamics unless the constraint in Eq.(10) is combined with it. The constraint in Eq.(10) may also be incorporated in the Lagrangian by the Lagrange multiplier method, giving a term $\lambda^a \partial^k A_k^a$ in the Lagrangian. Correspondingly, Eq.(8) will be replaced by

$$\dot{E}_k^a + \partial^l F_{kl}^a + gf^{abc}(A_0^b E_k^c + A^{bl} F_{kl}^c) - \partial_k E_0^a = 0 \tag{11}$$

where we have set $E_0^a = -\lambda^a$. Thus, the number of the equations (7),(9),(10) and (11) is equal to the number of the variables contained in the equations, including six canonical variables (A_k^a, E_k^a) and two Lagrange multipliers A_0^a and E_0^a for a given group index a. This fact shows self-consistence of the equations.

It is noted that in the Coulomb gauge, the canonical variables can only be the three-dimensional vectors A_k^a and E_k^a because the variable A_0^a has no its conjugate counterpart. In this case, we can only define the Poisson bracket through the three-dimensional vectors A_k^a and E_k^a and formulate the quantization Lorentz-non-covariantly. In order to perform the quantization

in a Lorentz-covariant manner, instead of the constraint in Eq.(10), it is suitable to choose the Lorentz gauge condition as the constraint

$$\varphi^a \equiv \partial^\mu A_\mu^a = 0 \tag{12}$$

Incorporating this constraint into the Lagrangian shown in Eq.(2) by the Lagrange multiplier procedure, we have

$$\mathcal{L} = E^{a\mu} \overset{\bullet}{A}{}_\mu^a + A_0^a C^a - E_0^a \varphi^a - \mathcal{H} \tag{13}$$

where

$$\pi_\mu^a = \frac{\partial \mathcal{L}}{\partial \overset{\bullet}{A}{}^{a\mu}} = \{ \begin{array}{c} F_{k0}^a = E_k^a, if \ \mu = k = 1,2,3; \\ -E_0^a, if \ \mu = 0. \end{array} \tag{14}$$

are the canonical momentum conjugate to A_μ^a,

$$C^a = \partial^\mu E_\mu^a + g f^{abc} A^{bk} E_k^c \tag{15}$$

and $\mathcal{H}$ was given in Eq.(5). As we see, in the Lorentz gauge, since the third term in Eq.(13) contains a time-derivative $\overset{\bullet}{A}{}_0^a$, the Lagrange multiplier A_0^a has its conjugate variable provided by the Lagrange multiplier $-E_0^a$. Thus, we have a Lorentz vector $E_\mu^a = (E_0^a, E_k^a)$ and the first term in Eq.(13) and the first term in Eq.(15) can be written in the Lorentz-invariant form. It is noted that the terms $E_0^a \overset{\bullet}{A}{}_0^a$, and $A_0^a \overset{\bullet}{E}{}_0^a$ appearing respectively in the first and second terms in Eq.(13) will be cancelled with each other in the action. Therefore, except for the third term, the sum of the other three terms in Eq.(13) actually is identical to the Lagrangian written in Eq.(2). In addition, we note, in order to get Lorentz-covariant results in later derivations, the second term in Eq.(15) may also be written in a Lorentz-invariant form

$$C^a = \partial^\mu E_\mu^a + g f^{abc} A^{b\mu} E_\mu^c \tag{16}$$

This is because the added term $g f^{abc} A_0^b E_0^c$ gives a vanishing contribution to the term $A_0^a C^a$ in Eq.(13) owing to the identity $f^{abc} A_0^b A_0^c = 0$.

Here we have necessity to discuss the equations of motion derived from the stationary condition of the action given by the Lagrangian in Eq.(13). These equations are

$$\overset{\bullet}{A}{}_\mu^a (x) - \frac{\delta H}{\delta E^{a\mu}(x)} + \int d^4x \{ A_0^b(y) \frac{\delta C^b(y)}{\delta E^{a\mu}(x)} - E_0^b(y) \frac{\delta \varphi^b(y)}{\delta E^{a\mu}(x)} \} - \varphi^a(x) \delta_{\mu 0} = 0 \tag{17}$$

$$\overset{\bullet}{E}{}^a_\mu(x) + \frac{\delta H}{\delta A^{a\mu}(x)} - \int d^4y\{A^b_0(y)\frac{\delta C^b(y)}{\delta A^{a\mu}(x)} - E^b_0(y)\frac{\delta\varphi^b(y)}{\delta A^{a\mu}(x)}\} - C^a(x)\delta_{\mu 0} = 0 \tag{18}$$

where the Hamiltonian is defined by [2],[5]

$$H = \int d^4x\mathcal{H}(x) \tag{19}$$

with $\mathcal{H}(x)$ being given in Eq.(5). In accordance with the definition of the Hamiltonian and expressions of $\varphi^a(x)$ and $C^a(x)$ as shown in Eqs.(12) and (16), the functional derivatives in Eqs.(17) and (18) are easily calculated. The results are

$$\frac{\delta\varphi^a(x)}{\delta A^b_\mu(y)} = \delta^{ab}\partial^\mu_x\delta^4(x-y) \tag{20}$$

$$\frac{\delta\varphi^a(x)}{\delta E^b_\mu(y)} = 0 \tag{21}$$

$$\frac{\delta C^a(x)}{\delta A^b_\mu(y)} = gf^{abc}E^{c\mu}(x)\delta^4(x-y) \tag{22}$$

$$\frac{\delta C^a(x)}{\delta E^b_\mu(y)} = [\delta^{ab}\partial^\mu_x - gf^{abc}A^{c\mu}(x)]\delta^4(x-y) \tag{23}$$

$$\frac{\delta H}{\delta A^a_\mu(x)} = [\partial^l_x F^a_{lk}(x) + gf^{abc}A^{bl}(x)F^c_{lk}(x)]\delta^{\mu k} \tag{24}$$

$$\frac{\delta H}{\delta E^a_\mu(x)} = E^a_k(x)\delta^{\mu k} \tag{25}$$

By making use of the above derivatives, one may find that Eqs.(17) and (18) will lead to the equations of motion shown in Eqs.(7) and (11) if we take $\mu = k$ in Eqs.(17) and (18) and the constraint equations written in Eq.(12) and in the following

$$C^a \equiv \partial^\mu E^a_\mu + gf^{abc}A^{b\mu}E^c_\mu = 0 \tag{26}$$

if we set $\mu = 0$ in Eqs.(17) and (18). It should be noted that in the derivation of the equations of $\mu = 0$, we have used the following equations

$$\overset{\bullet}{A}{}^a_0(x) - \frac{\delta H}{\delta E^a_0(x)} + \int d^4y[A^b_0(y)\frac{\delta C^b(y)}{\delta E^a_0(x)} - E^b_0(y)\frac{\delta\varphi^b(y)}{\delta E^a_0(x)}] = 0 \tag{27}$$

$$\overset{\bullet}{E}{}_0^a\,(x) + \frac{\delta H}{\delta A_0^a(x)} - \int d^4y[A_0^b(y)\frac{\delta C^b(y)}{\delta A_0^a(x)} - E_0^b(y)\frac{\delta \varphi^b(y)}{\delta A_0^a(x)}] = 0 \tag{28}$$

which always hold and appear to be identities. It is clear that the equations given in Eqs.(7),(11),(12) and (26) are sufficient to determine the eight canonical variables for a given group index which contain four dynamical variables, two constrained variables and two Lagrange multipliers. This indicates that the dynamics formulated in the Lorentz gauge is complete.

The canonical structure of the Lagrangian in Eq.(13) implies that the equations written in Eqs.(12) and (26) which are incorporated into the Lagrangian by the Lagrange multiplier method can only act as constraint conditions although there are time-derivatives in them. In contrast to those constraints given in Eqs.(9) and (10) which are stationary, these constraints are motional. Let us examine the consistency of these constraints along the line suggested by Dirac[6]. Taking derivatives of Eqs.(12) and (26) with respect to time and employing Eqs.(17) and (18), we obtain

$$\begin{aligned} & \{\varphi^a(x), H\} + \int d^4y A_0^b(y)\{C^b(y), \varphi^a(x)\} - \int d^4y E_0^b(y) \\ \times\{\varphi^b(y), \varphi^a(x)\} \quad = \quad & 0 \end{aligned} \tag{29}$$

$$\begin{aligned} & \{C^a(x), H\} + \int d^4y A_0^b(y)\{C^b(y), C^a(x)\} - \int d^4y E_0^b(y) \\ \times\{\varphi^b(y), C^a(x)\} \quad = \quad & 0 \end{aligned} \tag{30}$$

where the poisson bracket is defined as[5]

$$\{M, N\} = \int d^4x[\frac{\delta M}{\delta A_\mu^a(x)}\frac{\delta N}{\delta E^{a\mu}(x)} - \frac{\delta M}{\delta E_\mu^a(x)}\frac{\delta N}{\delta A^{a\mu}(x)}] \tag{31}$$

Based on this definition and utilizing the derivatives in Eqs.(20)-(25), it is not difficult to find

$$\{\varphi^a(x), \varphi^b(y)\} = 0 \tag{32}$$

$$\{C^a(x), C^b(y)\} = gf^{abc}C^c(x)\delta^4(x-y) = 0 \tag{33}$$

where Eq.(26) has been considered,

$$\{C^a(x), \varphi^b(y)\} = \partial_x^\mu (D_\mu^{ab}(x)\delta^4(x-y)) \tag{34}$$

where

$$D_\mu^{ab}(x) = \delta^{ab}\partial_\mu^x - gf^{abc}A_\mu^c(x) \tag{35}$$

$$\{\varphi^a(x), H\} = -\partial^k E_k^a \tag{36}$$

and

$$\{C^a(x), H\} = 0 \tag{37}$$

It is emphasized that the nonvanishing of the Poisson bracket in Eq.(34) implies that the equations (29) and (30) are solvable to the Lagrange multipliers A_0^a and E_0^a. Substitution of the above poisson brackets into Eqs.(29) and (30) yields

$$D_\mu^{ab}\partial^\mu E_0^b = 0 \tag{38}$$

and

$$D_\mu^{ab}\partial^\mu A_0^b = \partial^k E_k^a \tag{39}$$

These are the second order differential equations for the variables A_0^a and E_0^a.

For later purpose, it is useful to consider solutions of the equations (12) and (26). Noticing the decomposition $A^{a\mu} = A_T^{a\mu} + A_L^{a\mu}$ where $A_T^{a\mu}$ and $A_L^{a\mu}$ are respectively the transverse and longitudinal components of the vector $A^{a\mu}$ and the transversality condition $\partial^\mu A_{T\mu}^a = 0$, Eq.(12) may be written as

$$\partial^\mu A_{L\mu}^a = 0 \tag{40}$$

Its solution, as is well-known , is

$$A_{L\mu}^a = 0 \tag{41}$$

Similarly, when the decomposition $E^{a\mu} = E_T^{a\mu} + E_L^{a\mu}$ is inserted into Eq.(26), noticing the transversality $\partial^\mu E_{T\mu}^a = 0$ and the solution in Eq.(41), Eq.(26) will be reduced to

$$\partial^\mu E_{L\mu}^a + gf^{abc}A_T^{b\mu}(E_{T\mu}^c + E_{L\mu}^c) = 0 \tag{42}$$

Using the expression

$$E_{L\mu}^a = \partial_\mu Q^a \tag{43}$$

where Q^a is a scalar function, we obtain from Eq.(42)

$$K^{ab}(x)Q^b(x) = \omega^a(x) \tag{44}$$

where

$$K^{ab}(x) = \delta^{ab}\Box_x - gf^{abc}A_T^{c\mu}(x)\partial_\mu^x \tag{45}$$

and

$$\omega^a(x) = gf^{abc}E_{T\mu}^b A_T^{c\mu} \tag{46}$$

With the aid of the Green function(the ghost particle propagator) $D^{ab}(x-y)$ which satisfies the equation

$$K^{ac}(x)D^{cb}(x-y) = \delta^{ab}\delta^4(x-y) \tag{47}$$

the solution of Eq.(44) is found to be

$$Q^a(x) = \int d^4y D^{ab}(x-y)\omega^b(y) \tag{48}$$

which is a function of the transverse vectors $A_T^{a\mu}$ and $E_T^{a\mu}$. Thus, the function $E_L^{a\mu}$ may be expressed in terms of the $A_T^{a\mu}$ and $E_T^{a\mu}$. With the longitudinal components of the vectors A_μ^a and E_μ^a being determined by the constraint equations, the transverse components $A_T^{a\mu}$ and $E_T^{a\mu}$ of the vectors $A^{a\mu}$ and $E^{a\mu}$ act as the independent canonical variables. By employing the solutions of the constraint equations, obviously, the Hamiltonian density denoted in Eq.(5) may be represented via the independent variables

$$\mathcal{H}^*(A_{T\mu}^a, E_{T\mu}^a) = \mathcal{H}(A_\mu^a, E_\mu^a)_{|\varphi^a=0,c^a=0} \tag{49}$$

Now, we are in a position to formulate the quantization performed in the Hamiltonian path-integral formalism for the massless non-Abelian gauge field. According to the general principle of the quantization, we should at first construct the generating functional of Green's functions via the independent variables

$$\begin{aligned} Z[J] \quad = \quad & \frac{1}{N}\int D(A_{T\mu}^a, E_{T\mu}^a)\exp\{i\int d^4x[E_T^{a\mu}\ \dot{A}_{T\mu}^a \\ & -\mathcal{H}^*(A_T^{a\mu}, E_T^{a\mu}) + J_T^{a\mu}A_{T\mu}^a]\} \end{aligned} \tag{50}$$

In order to express the generating functional in terms of the full vectors A^a_μ and E^a_μ, it is necessary to introduce the δ-functional $\delta[A^{a\mu}_L]\delta[E^{a\mu}_L - E^{a\mu}_L(A^{a\mu}_T, E^{a\mu}_T)]$ into the functional. It is easy to prove that[2],[5]

$$\delta[A^{a\mu}_L]\delta[E^{a\mu}_L - E^{a\mu}_L(A^{a\mu}_T, E^{a\mu}_T)] = \det M\delta[\varphi^a]\delta[C^a] \tag{51}$$

where M is the matrix whose elements are

$$M^{ab}(x,y) = \{C^a(x), \varphi^b(y)\} \tag{52}$$

which were given in Eq.(34). Upon inserting Eq.(51) into Eq.(50) and using the Fourier representation of the δ-functional

$$\delta[C^a] = \int D(\eta^a/2\pi)e^{i\int d^4x\eta^a c^a} \tag{53}$$

We have

$$\begin{aligned} Z[J] &= \frac{1}{N}\int D(A^a_\mu, E^a_\mu, \eta^a)\det M\delta(\partial^\mu A^a_\mu) \\ &\quad \times\exp\{i\int d^4x[E^{a\mu}\dot{A}^a_\mu + \eta^a C^a - \mathcal{H}(A^a_\mu, E^a_\mu) + J^{a\mu}A^a_\mu]\} \end{aligned} \tag{54}$$

Noticing the expression given in Eq.(15), we see, in the above exponent, there is a E^a_0-related term $E^a_0(\partial_0 A^a_0 - \partial_0\eta^a)$. It allows us to perform the integration over E^a_0, giving a δ-functional $\delta[\partial_0 A^a_0 - \partial_0\eta^a] = \det|\partial_0|^{-1}\delta[A^a_0 - \eta^a]$. The determinant $\det|\partial_0|^{-1}$, as a constant, may be put in the normalization constant N. The δ-functional $\delta[A^a_0 - \eta^a]$ will disappears when the integration over η^a is carried out. Thus, considering the expressions given in Eqs.(5),(6) and (15), we can write

$$\begin{aligned} Z[J] &= \frac{1}{N}\int D(A^a_\mu, E^a_k)\det M\delta[\partial^\mu A^a_\mu]\exp\{i\int d^4x \\ &\quad \times[-\frac{1}{2}(E^a_k)^2 + E^a_k F^{a0k} - \frac{1}{2}F^{akl}F^a_{kl} + J^{a\mu}A^a_\mu]\} \end{aligned} \tag{55}$$

After calculating the Gaussian integral over E^a_k, we arrive at

$$\begin{aligned} Z[J] &= \frac{1}{N}\int D(A^a_\mu)\det M\delta[\partial^\mu A^a_\mu]\exp\{i\int d^4x \\ &\quad -\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} + J^{a\mu}A^a_\mu]\} \end{aligned} \tag{56}$$

When we employ the familiar expression[1]

$$\det M = \int D(\overline{C}^a, C^a) e^{i\int d^4x d^4y \overline{C}^a(x) M^{ab}(x,y) C^b(y)} \tag{57}$$

where $\overline{C}^a(x)$ and $C^a(x)$ are the mutually conjugate ghost field variables and the following limit for the Fresnel functional

$$\delta[\partial^\mu A^a_\mu] = \lim_{\alpha\to\infty} C[\alpha] e^{-\frac{i}{2\alpha}\int d^4x (\partial^\mu A^a_\mu)^2} \tag{58}$$

where $C[\alpha] = \prod_x (\frac{i}{2\pi\alpha})^{\frac{1}{2}}$ and supplementing the external source terms for the ghost fields, the generating functional will finally be written in the form

$$\begin{aligned} Z[J,\overline{\xi},\xi] &= \frac{1}{N}\int D(A^a_\mu, \overline{C}^a C^a) \exp\{i\int d^4x[\mathcal{L}_{eff} \\ &\quad + J^{a\mu} A^a_\mu + \overline{\xi}^a C^a + \overline{C}^a \xi^a]\} \end{aligned} \tag{59}$$

where

$$\mathcal{L}_{eff} = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} - \frac{1}{2\alpha}(\partial^\mu A^a_\mu)^2 - \partial^\mu \overline{C}^a D^{ab}_\mu C^b \tag{60}$$

which is the effective Lagrangian for the system under consideration. In Eq.(59), the limit $\alpha \to 0$ is implied. Nevertheless, this limit is unnecessary if we work in general gauges where $\alpha \neq 0$. In these gauges, the Lorentz condition will be extended to

$$\partial^\mu A^a_\mu - \alpha E^a_0 = 0 \tag{61}$$

For this case, it is easy to verify that the poisson bracket $\{C^a(x), \varphi^b(y)\}$ is still given by Eq.(34) and hence the matrix M remains unchanged. Therefore, when the δ-functional $\delta[\partial^\mu A^a_\mu]$ in Eq.(56) is replaced by $\delta[\partial^\mu A^a_\mu - \alpha E^a_0]$ and then acting on Eq.(56) with the integration operator $\int D(E^a_0) e^{-\frac{i}{2\alpha}(E^a_0)^2}$, we still obtain the generating functional given in Eqs.(59) and (60) with the α being arbitrary. This generating functional is completely the same as given by the Faddeev-Popov approach[1].

Up to the present, the Lorentz-covariant quantization in the Hamiltonian path-integral formalism has been achieved by the novel procedure proposed in this paper. The essential feature of the procedure is that the Lorentz gauge condition, as a necessary constraint. is introduced from the beginning and according to the general procedure established well in mechanics

for constrained systems, it may be incorporated into the Yang-Mills Lagrangian by the Lagrange multiplier method. In this way, it is found that the four-dimensional vector potential A^a_μ has its four-dimensional conjugate counterpart. These mutually conjugate canonical variables allow us to define the poisson bracket and perform the quantization in a Lorentz-covariant manner. Obviously, the procedure presented in this paper is more general than the ordinary one because the procedure is suitable to quantize the gauge field in any gauge. Moreover, by this procedure, one does not need to make the distinction between the primary constraint and the second one as well as between the first-class constraint and the second-class one. The necessary constraints may be chosen from the physical requirement for the constrained system under consideration.

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Reference

[1] L. D. Faddeev and V. N. Popov, Phys. Lett. B25, 29 (1967).

[2] L. D. Faddeev, Theor. Math. Phys. 1, 1 (1970).

[3]E. S. Abers and B. W. Lee, Phys. Rep. C9, 1 (1973).

[4]P. Senjanovic, Ann. Phys. (N. Y.) 100, 227 (1976).

[5]L. D. Faddeev and A. A. Slavnov, Gauge fields: Introduction to Quantum Theory, The Benjamin Commings Publishing Company Inc. (1980).

[6]P. A. M. Dirac, Lectures on Quantum Mechanics, Belfer Graduate School, Yeshiva Univ. Press, New York (1964).